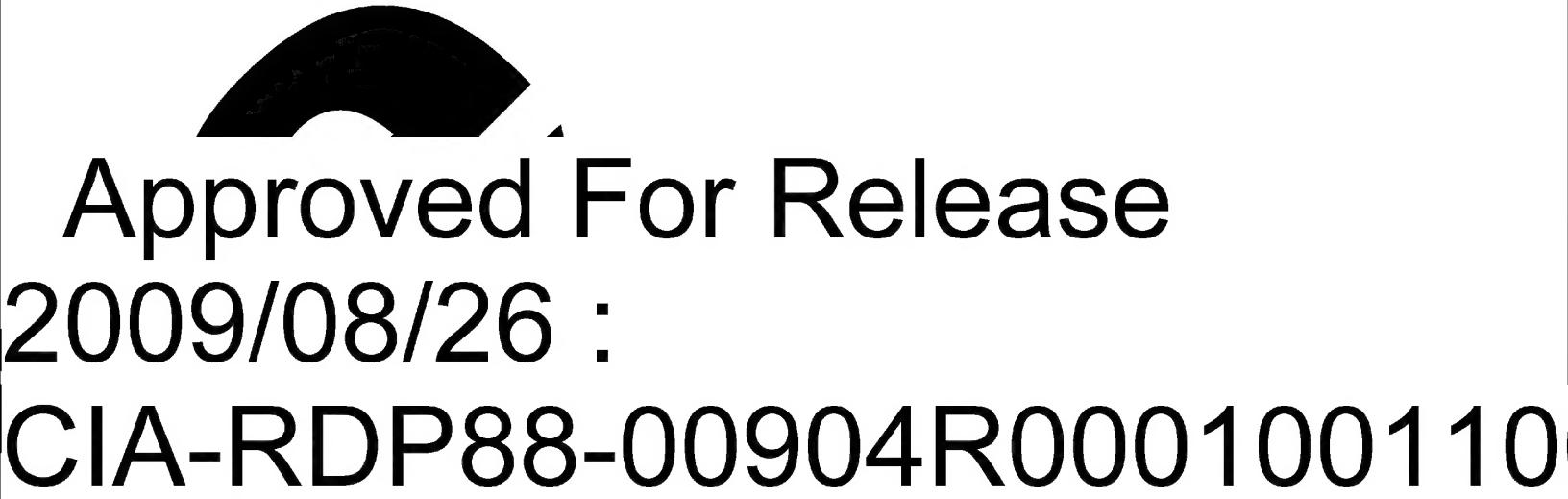


Approved For Release STAT
2009/08/26 :
CIA-RDP88-00904R000100110

A large, solid black redaction mark is positioned in the center of the page. It is a thick, irregular shape that covers a significant portion of the upper half of the page, obscuring the text above it.

A large, solid black redaction mark is positioned in the center of the page. It is a thick, irregular shape that covers a significant portion of the lower half of the page, obscuring the text below it.



Third United Nations
International Conference
on the Peaceful Uses
of Atomic Energy

A/CONF.28/P/354
USSR
May 1964
Original: RUSSIAN

Confidential until official release during Conference

CHOICE OF SOME OPTIMUM CHARACTERISTICS FOR
THE REACTOR CONTROL SYSTEM.

V.V.Orlov, V.S.Andreyanov, E.I.Grishanin, L.I.Isakova,
A.G.Kalashnikov, J.G.Pashkin, G.I.Toshinsky, V.V.Chekunov.

The control system compensating the excess reactivity can essentially influence upon the physical and operating reactor characteristics. This influence increases when compensating big reactivity excess. That is why the problems of choosing reactor control system optimum characteristics along with the problems of principal reactor characteristics optimization are of a great interest. In the first part of this report some questions of optimum burnable poison characteristics choice are considered. The second part describes the choice of an optimum resonance absorber mixture for control rods.

PART I.
BURNABLE POISONS.

In power reactors along with mechanical control devices for compensating reactivity excess burnable poisons are widely used now (for instance see [1]). Some problems of the calculation theory and of the use of burnable poisons have been considered in [2, 3, 4, 5]. These problems of thermal reactors are discussed in [2][3], some particular problems for intermediate reactors being studied in [4]. The method of burnable poison calculation is discussed as a more general case in [5].

The main purpose of the burnable poison application is to decrease the excess reactivity compensating by the mechanical control devices. That is to say the choice of optimum burnable poison is the first of all the choice of absorption materials and a means of distribution them to provide the best agreement between the reactivity loss

caused by fuel depletion and poison burn-up. Therefore the residual poisoning caused by non burnt-out absorbers and other isotopes which were present in the original material and were also formed after burning out must be minimized.

To minimize the residual reactor poisoning due to the non-burntup absorber at the end of the core lifetime the poison burn-up rate must be much more than that of fuel depletion.

If such a poison can be chosen properly for the reactor under consideration the homogeneous poison distribution over the reactor results in the rapid poison burn-up and the premature reactivity release. In this case the heterogeneous poison distribution turns out to be more effective because of decreasing neutron flux (self-shielding effect) in the absorber which results in decreasing the absorber burn-up rate that in its turn permits to reduce the reactivity mismatch caused by different burn-up laws of poison and fuel.

If such poison can't be chosen properly for reactor under consideration the heterogeneous distribution results in greater reactivity loss caused by the big residual non burnt-up poison to the end of the core life-time and therefore homogeneous distribution may be more expedient .

A. Homogeneous poison distribution.

The homogeneous poison distribution is considered in detail in [5]. The plots given there permit to define the maximum reactivity mismatch and residual poisoning due to non burnt-up poison to the end of the core life-time if poison and fuel life-time are known.

B. Heterogeneous poison distribution.

a). Plate-type absorber. Heterogeneous poison distribution in plane geometry is considered in [5] and some criteria are found which permit to determine the poison char-

acteristics providing the maximum reduction of the number of mechanical control devices (or their efficiency).

According to the method given in [5] some calculations were carried out to determine the region where the plate-type absorbers are the most effective ones. The perturbation theory[6] is used for calculating the reactivity change due to poison burn-up, fuel depletion and poisoning. The approximate formula derived in [5] to calculate reactivity change as a function of these processes is

$$\rho_n(y)/\rho_n(0) = z(y)[1+\gamma\beta_0]/[1+\gamma\beta_0 z(y)][1-\frac{y}{x_r}] \quad (1.1)$$

and

$$\rho_{n,un}(y)/\rho_{n,un}^0 = y(1-\frac{1}{x_r})/(1-\frac{y}{x_r}) \quad (1.2)$$

Definition of symbols:

$\rho_n(y)$ - reactivity introduced by an absorber into the reactor at time y .

y - dimensionless time expressed in endurance fractions
 $y = \frac{t}{T}$

t - reactor life-time scaled to rated power.

T - endurance.

β_0 - optical width of absorber plate at the beginning of the reactor life-time $\beta_0 = N_n(0)G_c \Delta$

$N_n(0)$ - absorber nucleus concentration in the plate at the beginning of the reactor life-time.

G_c - microscopic absorption cross section averaged over the neutron spectrum.

Δ - thickness of absorber plate.

γ - parameter giving the best approximation of the plate self-shielding factor dependence $f(\beta)$ as a function of its optical width, $f(\beta) = 1/(1+\gamma\beta)$.

x_r - dimensionless fuel life-time $x_r = 1/T \int G_c(u) \Phi(u) du$

G_c - fuel microscopic absorption cross section.

$\Phi(u)$ - neutron flux in the reactor at the beginning of the reactor life-time, also normalized to the rated power.

$\rho_{\text{run}}(y)$ - reactivity excess to compensate fuel depletion and poisoning at time y .

ρ_{run}^0 - initial reactivity excess to compensate fuel depletion and poisoning.

$Z(y)$ - relative concentration of absorber nuclei $Z = N_n(y)/N_n(0)$

$N_n(y)$ - the absorber nucleus concentration averaged over plate thickness at time y .

$Z(y)$ - was determined from equation [5] (see also [4])

$$Z e^{-\gamma \beta_0 (1-Z)} = \left(1 - \frac{y}{X_r}\right)^{\frac{X_r}{X_n}} \quad (1.3)$$

X_n - dimensionless absorber life-time $X_n = 1/T \int G_c^n(u) \Phi_0(u) du$

Formulae (1.1), (1.2), (1.3) are obtained with the some assumptions which essentially simplify these calculations:

- Neutron flux and importance don't depend on the coordinates and are equal to the values averaged over the reactor.
- Neutron spectrum don't vary with time $\Phi(u, y) = S(y) \Phi_0(u)$ where $S(y) = 1/(1 - \frac{y}{X_r})$
- The initial fuel and poison distribution over the core does not depend on coordinates.
- The fuel is distributed homogeneously (or in case of heterogeneous fuel distribution the self-shielding factor is time-independant).
- New fissionable isotopes are not generated in the reactor.
- The absorber sample size is small in comparison with the transport free part of surrounding medium so that the external self-shielding effect can be neglected.
- The exact expression for the self-shielding factor is approximated with function $f(\beta) = 1/(1 + \gamma \beta)$
- The cross section as a function of lethargy is represented by $\sigma(u) = \text{const}$ at $u \geq u_0$ and $\sigma_c(u) = 0$ at $u < u_0$.

Expressions for determinating the reactivity mismatch and the residual poisoning as a result of no burnable poison residue at the end of the core life-time can be obtained from the equation (1.1) and (1.2)

$$\rho_n(y)/\rho_{n,un}^o = \frac{1 - \rho_n(y)/\rho_n(0)}{1 - \rho_n(1)/\rho_n(0)} m - \rho_{n,un}(y)/\rho_{n,un}^o (1.4)$$

$$\rho_n(1)/\rho_{n,un}^o = m \frac{\rho_n(1)/\rho_n(0)}{1 - \rho_n(1)/\rho_n(0)} \quad (1.5)$$

where $m = [\rho_n(0) - \rho_n(1)]/\rho_{n,un}^o$ - the ration of reactivity released by burnable poison at the end of the core life-time to initial reactivity excess compensating the fuel depletion and poisoning.

$\rho_p(y)$ - reactivity mismatch at time y .

$\rho_n(1)$ - residual poisoning at the end of the core life-time.

In [5] the maximum reduction of mechanical control devices is shown to take place at such values of the plate optical width β_o when maximum mismatch $[\rho_n(y)/\rho_{n,un}^o]_{\max}$ is a fraction of initial reactivity excess for fuel depletion and poisoning (1.5) which are compensated by the mechanical control devices. In this case the reactivity mismatch has to vanish once during the core life-time.

The calculations were made for $X_p = 2.76$ and ω_p^o (the fraction of fuel depletion is $1/X_p = 0.36$ and 0) within $X_n \in 0 \div 0.3$. This range covers the most interesting practical cases when the plate-type absorber may be accepted. The initial optical width β_o of the absorber plate providing the maximum reduction of mechanical control devices is plotted as a function of absorber life-time X_n in Fig. 1. Fig. 2 shows the maximum reactivity mismatch and non burnable residual poisoning calculated as a function of absorber life-time for β_o values determined above.

The range where the use of the plate-type absorber becomes more effective is shown in Fig. 2 to be within $X_n (0.1 \div 0.25)$. The residual poisoning becomes too large when $X_n > 0.25 \div 0.30$, and the maximum reactivity mismatch rising very quickly up to $1/(1 + \sqrt{1 - \frac{1}{X_p}})$ at $X_n = 0$ when $X_p \leq 0.05 \div 0.1$.

The use of too large cross section absorbers ($X_n \ll 1$) provides their complete burn-up but if the absorber is

distributed as plates the big reactivity mismatch is observed. The mismatch can be decreased by using the combination of different optical width plates.

b). Rod-type absorbers.

The simplest solution is to distribute the large cross section absorbers as cylindrical rods* with optical width $\beta_o \gg 1$ that allows simultaneously to get small residual poisoning and small reactivity mismatch.

In this case the rod will be burnt up only within a narrow surface layers. The diameter of rod "blackness" and hence the releasing reactivity will vary according to the law which is similar to that one required for compensating fuel depletion and poisoning and the contradiction between the reactivity mismatch and the residual poisoning vanishes with burn-up taking place practically completely at very large cross sections. The absorbers with surface burn-up in contrast to the absorbers with temperate self-shielding may be figuratively called burnt around absorbers.

To find out physical features of the burnt-around absorbers it is usefull to consider the limiting case of absorber with the very large absorption cross section Θ represented as $\Theta_c^n = \infty$ at $u > u_0$ and $\Theta_c^n = 0$ at $u < u_0$. Thus the neutrons will be absorbed in infinite thin layers. The law of rod "blackness" radius change as a function of time can be established on the basis of the following considerations. The number of neutrons absorbed during time Δt is equal to $\frac{\Phi(t)}{4} 2\pi r^2 \Delta t$, where $\Phi(t)$ is integrated neutron flux within energy range where the absorption cross section differs from zero, r - rod "blackness" radius. On the

* The cylinder cross section may have a convex polygonal form where circle can be inscribed.

Here and below the assumptions made above are used.

other hand the corresponding change of the rod volume due to burn-up at the rod surface is equal to $2\pi r dr$. Thus the following relation can be written

$$\frac{\Phi(t)}{4} 2\pi r dt = 2\pi r N_n^o dr \quad (1.6)$$

After integrating the relation (1.6) in the corresponding limits we obtain

$$\int_0^t \Phi(t') dt' = 4RN_n^o (1 - \frac{r}{R})$$

The rod "blackness" radius as well as its absorption capacity is seen to decrease approximately as a linear function of time giving a small reactivity mismatch. Let us evaluate the reactivity mismatch arising due to incomplete correspondence between fuel and poison burn-up laws. With $\Phi(t) = S(t) \int \Phi_0(u) du$ (where $S(t) = 1/(1 - \frac{y}{X_r})$) and the requirement of the complete absorber burn up to time $y = y_0$ of the core lifetime the expression of rod radius change can be written as

$$1 - r(y)/R = \ln(1 - \frac{y}{X_r}) / \ln(1 - \frac{y_0}{X_r}) \quad (1.7)$$

the initial rod optical width being $\rho_0 = -\frac{1}{2} \frac{X_r}{X_n} \ln(1 - \frac{y_0}{X_r})$ it is here convenient to take the absorber cross section as finite one.

Using the expression (1.7) we obtain the relation for reactivity release during burning around of a rod.

$$1 - \frac{\rho_n(y)}{\rho_n(0)} = 1 - r(y) S(y)/R = 1 - \left[1 - \frac{\ln(1 - \frac{y}{X_r})}{\ln(1 - \frac{y_0}{X_r})} \right] / \left(1 - \frac{y}{X_r} \right) \quad (1.8)$$

at $y \leq y_0$.

$$\text{and } 1 - \frac{\rho_n(y)}{\rho_n(0)} = 1 \quad \text{at } y_0 < y \leq 1$$

From the relations (1.8) and (1.2) we find the expression for reactivity mismatch under condition of the burnup around absorber compensating (at the beginning of core lifetime) the fraction m of total reactivity excess for burnup and poisoning.

$$\frac{\rho_n(y)}{\rho_n(0)} = m \left\{ 1 - \left[1 - \frac{\ln(1 - \frac{y}{X_r})}{\ln(1 - \frac{y_0}{X_r})} \right] / \left(1 - \frac{y}{X_r} \right) \right\} - y \left(1 - \frac{1}{X_r} \right) / \left(1 - \frac{y}{X_r} \right) \quad (1.9)$$

at $y \leq y_0$.

$$\text{and } \frac{\rho_n(y)}{\rho_{n, \text{max}}} = m - y \left(1 - \frac{1}{X_r} \right) / \left(1 - \frac{y}{X_r} \right) \quad \text{at } y_0 < y \leq 1$$

Setting equal to zero the y derivative from the expression (1.9) we obtain the relation for the maximum reactivity mismatch within the range

$$\frac{\rho_p(y_{\max})}{\rho_{p,un}^0} = (x_r - 1 + m) \left\{ 1 + \left[e^{\frac{x_r-1+m}{m}} \left(1 - \frac{y_0}{x_r} \right)^{\frac{x_r-1+m}{m}} \ln \left(1 - \frac{y_0}{x_r} \right)^{-1} \right] \right\} \quad (1.10a)$$

$y_{\max} = x_r \left[1 - e^{\left(1 - \frac{y_0}{x_r} \right) \frac{x_r-1+m}{m}} \right]$

When $y = y_0$ the reactivity mismatch is

$$\frac{\rho_p(y_0)}{\rho_{p,un}^0} = m - y_0 \left(1 - \frac{1}{x_r} \right) / \left(1 - \frac{y_0}{x_r} \right) \quad (1.10b)$$

and at the end of core life-time ($y = 1$)

$$\frac{\rho_p(1)}{\rho_{p,un}^0} = - (1 - m) \quad (1.10c)$$

The maximum reactivity mismatch can be easily shown from (1.10a) as being always negative. This means that at the beginning of the core life-time the burnable poison can compensate not the total reactivity excess for burn up and poisoning but only its fraction m . The residual part of the reactivity excess $(1 - m)$ must be evidently compensated by mechanical control devices and $(1 - m)$ has not to be smaller than maximum reactivity mismatch. With the aim of reducing the use of mechanical control devices it should be make $1 - m = - \frac{\rho_p(y_{\max})}{\rho_{p,un}^0}$ and the following equation connecting y_0 and m results

$$1 = - e^{\frac{x_r}{m} \left(1 - \frac{y_0}{x_r} \right)^{\frac{x_r-1+m}{m}}} \ln \left(1 - \frac{y_0}{x_r} \right) \quad (1.11)$$

The reactivity mismatch at $y = y_0$ may be positive, negative or equal to zero depending on the value of parameter y_0 (1.10b). Optimum case can be shown as a case when the mismatch at $y = y_0$ is equal to zero. From there we obtain the second equation connecting the values y_0 and m .

$$m = y_0 \left(1 - \frac{1}{x_r} \right) / \left(1 - \frac{y_0}{x_r} \right) \quad (1.12)$$

From (1.11) and (1.12) the equation to determine the optimum value m is got

$$\alpha \ln \frac{\alpha}{\alpha-1} = 1 + \ln(\alpha + M) + \ln \ln \frac{\alpha}{\alpha-1} \quad (1.13)$$

where

$$\alpha = \frac{x_r - 1 + m}{m} \quad , \quad M = \frac{1 - m}{m}$$

The results of calculations plotted in Fig.3 show the maximum reactivity mismatch ($1-m$) to be considerably smaller in case when the burnable poison with the large absorption cross section is distributed in form of rods than when it is distributed in form of plates. So the use of burnable poisons with the infinite absorption cross section results in minimum number of necessary mechanical control devices without reactivity loss due to non burnt up poison residue. The transition from ideal model to practical absorbers sets some additional problems. We shall discuss the following ones.

1). To clear up the burn up kinetics of absorbes with finite cross section $\bar{\sigma}_c$ we shall consider the problems of determining the absorber distribution over the semi-infinite medium; the neutron flux incident at its surface is \bar{J} neutron $\text{cm}^{-2} \text{sec}^{-1}$. If the initial absorber distribution was uniform so by time

$$t \text{ it will be } N_n(x,t) = N_n^0 \left[1 + (1 - e^{-\bar{\sigma}_c t}) \exp(\bar{\sigma}_c t - \bar{\sigma}_c N_n^0 x) \right]^{-1} \quad (1.14)$$

With the use of this formula the absorption capacity can be shown to decrease slowly during time $t \leq \bar{\tau} = 1/\bar{\sigma}_c$. At time $t \sim \bar{\tau}$ the $N(x)$ distribution front of the width $\Delta x \sim \frac{1}{N_n^0 \bar{\sigma}_c}$ has formed and it is moving with the constant rate $\frac{dx}{dt} = \frac{\bar{J}}{N_n^0 \bar{\sigma}_c}$ inside the absorber (see Fig.4)

Since the value $\bar{\sigma}_c$ being finite it could be expected the deviation from linear dependance $\zeta(t)$ during time $\bar{\tau} \sim \frac{1}{2\bar{\sigma}_c}$ at the first operation period (the incident flux being isotropic the front width decreases two times as compared to the normal incident flux. Thereby $\zeta \approx \frac{\bar{J}}{2}$). This deviation from formula (1.7) will also take place at the final stage of burn up, the optical rod width having decreased up to the value $\beta \sim 1$, and consequently, the burn up rate will increase. The summary time turns out to be equal to $\sim \bar{\tau} = \frac{1}{\bar{\sigma}_c}$ and so that it would not exceed for example 10% of the core lifetime it is necessary that $\bar{\tau} \leq 0.1 T$, or $\frac{1}{\bar{\sigma}_c} \leq \frac{0.1}{\bar{J}}$. This estimation shows that the isotopes of $\text{G}_c \geq 4\text{G}_c$ can be used

as burnt-around poisons.

The cross sections greatly exceeding cross sections of fissionable isotopes exist only in the thermal region when the energy of one of the resonances is near to zero. That is why the burnt-around poisons in practice can be used only for thermal reactors, of stable isotopes only Cd¹¹³, Sm¹⁴⁹, Gd¹⁵⁵ and Gd¹⁵⁷ satisfying the above condition.

2). The volume absorber burn up due to neutron absorption in energy region where the cross sections are not large leads also to the change of the burn up low. In the thermal region very large absorption cross sections caused by low-lying resonances decrease with the energy increase according to the Bruegh-Wigner formula ($\bar{\sigma}_c \sim \frac{1}{E^{5/2}}$ law). If the spectrum of neutron incident onto the rod is $\Phi(E)$ the number of neutrons absorbed by this rod at the energy $E > E_{rp}$ (their free path being $\lambda(E) > \lambda = 2R$) is

$$A_v \approx \int_{E_{rp}}^{\infty} \Phi(E) V N_n^o \bar{\sigma}_c(E) dE \approx V N_n^o \bar{\sigma}_c(E_{rp}) \int_{E_{rp}}^{\infty} \frac{(E_{rp})^{\frac{1}{2}}}{E^{\frac{5}{2}}} dE = \frac{2}{5} V N_n^o \bar{\sigma}_c(E_{rp}) E_{rp} \Phi(E)$$

and $N_n^o \bar{\sigma}_c(E_{rp}) = \frac{1}{2R}$ so that finally

$$A_v \approx \frac{5}{10} E_{rp} \Phi(E_{rp}) = \frac{5}{10} \Phi_r \frac{\Sigma_c}{\Sigma_s}$$

The spectrum is assumed here to be Fermi spectrum. The number of neutrons absorbed as the surface is $A_s = \frac{5}{4} \int_{E_{rp}}^{\infty} \Phi(E) dE$ therefore the volume absorption fraction is $A_v/A_s \approx \frac{2}{5} \frac{\Sigma_c}{\Sigma_s}$ that usually constitutes in the thermal reactors the value ≈ 0.1 .

The value A must be supplemented with the absorption of resonance region which is small for the isotopes mentioned above. The volume absorption effect leads to some increase of reactivity release rate with burning up.

3). Neutron flux depression near the rod (external shielding) leads to decreasing of burn-up rate at the beginning of core life-time. This effect becomes negligible with using of small diameter rods.

The absorber distribution over a core.

The above mentioned consideration permit to choose cont-
354

rod rods (their number, diameter and absorber density) providing a small change of the neutron multiplication factor in some reactor volume if the fuel burn-up reached in this volume to the end of the core life-time is known. As power distribution over the reactor depends in its turn on the rod distribution the nonlinear problem arise and they can be solved by the iterative method.

In ideal case we can reduce this problems to following nonlinear equation. Suppose that a) the absorber provides constant multiplication factor K_{∞} as a function of time in every point over the reactor; b) the absorption probability at point \vec{r} of a neutron born in point \vec{r}' is $P(\vec{r}, \vec{r}')$ and does not vary during the core life-time (for instance it takes place in thermal water-moderated reactors where $L^2 \ll \tau$). Then the neutron source density $Q(\vec{r})$ (or power density proportional to it) is described by the following equation

$$Q(\vec{r}) = K_{\infty}(\vec{r}) \int Q(\vec{r}') P(\vec{r}, \vec{r}') d\vec{r}' \quad (1.15)$$

The value K_{∞} may be expressed in term of fuel burn-up reached at point \vec{r} $\frac{1}{X_r(\vec{r})} \sim Q(\vec{r})$ at the end of the core life-time. So we obtain the nonlinear equation for $Q(\vec{r})$

$$Q(\vec{r}) = K_{\infty}(Q(\vec{r}), \vec{r}) \int Q(\vec{r}') P(\vec{r}, \vec{r}') d\vec{r}' \quad (1.16)$$

Expanding $Q(\vec{r})$ at point \vec{r}' and taking into account only three terms of this expansion we have at $P=P(|\vec{r}-\vec{r}'|)$

$$\nabla^2 Q - \frac{K_{\infty}(Q, \vec{r}) - 1}{K_{\infty}(Q, \vec{r}) M^2} Q = 0 \quad (1.17)$$

where $M^2 = \int P(\vec{r}') r^4 dr / \int P(r) r^2 dr$

Boundary conditions for Q can be written with the help of reflector albedo. With known power distribution Q and with burn-up distribution proportional to it the law of absorber distribution over the core which provides constant "K" at every point \vec{r} may be easily found. This distribution provides not only constant reactivity during the whole reactor life-time but also uniform power distribution over the

reactor* with its simultaneous flattening (compared to uniform control rod distribution).

II. Resonance absorbers for control rods.

The absorption cross section energy dependence of the materials is especially important for control rods. The use of resonance absorbers permits within some limits to get the most profitable absorption cross section energy dependence for the reactor under consideration with regard as to increasing of the efficiency so to decreasing of neutron flux depression near the rod. The latter is the most important for thermal reactors.

To increase the rod efficiency or to decrease neutron flux depression near the rod it is of great interest to use a mixture of different resonance absorbers. In this case the total resonance absorption can be more than the absorption of a component with the greatest resonance integral due to the reduction of every component resonance self-shielding. For instance if there are "n" absorbers of narrow and strong resonances every one of which differs from one another only by resonance energy the maximum value of mixture resonance integral will be \sqrt{n} times more than resonance integral of a single absorber. The practical absorbers differ from one another by nucleus density, resonance energy, resonance parameters and so on. Furthermore the absorption by weak resonances and by "smooth" part of the absorption cross section constitutes the considerable part of the total absorption. That is why the optimum mixture composition depends not only on the properties of chosen absorbers but also on the form of neutron spectrum, temperature and so on.

* The problems of control system providing the flat time-independent distribution over the reactor was set and solved by Sharapov V.N.

A. Determination of optimum resonance absorber composition to obtain the maximum rod efficiency.

According to the perturbation theory [6] the rod efficiency is

$$\rho = \frac{1}{4HD} \int dv \int du W(u) \sum_{i=1}^n N_i G_c^i(u) f_i(d, N_k, G_c^k(u)) \quad (2.1)$$

where $W(u)$ - product of neutron flux with energy "u" and the importance of these neutrons, f_i - self-shielding factor of its element absorption cross section which depends on all the values of G_c^k and N_k ; $4HD$ - importance of fission neutrons; d - rod diameter; n - a number of mixture components.

To obtain the maximum efficiency of the rod of given size which is made of the resonance absorber mixture it is necessary to determine the maximum of expression (2.1) with $\sum_i V_i = 1$ (V_i - a volume part of i -th component). This problem is the conventional extremum one and is solved by means of undefined Lagrangian multipliers method. According to this method the absolute extremum of function $F = \rho + \lambda L$, where $L = \sum_i V_i - 1$ and λ is an undefined multiplier, must be found. Necessary conditions of the function F extremum

$$\frac{\partial F}{\partial N_i} = 0, \quad \frac{\partial F}{\partial \lambda} = L = 0 \quad (2.2)$$

give us the system of $(n+1)$ differential equations with unknown quantities N_i and λ . On this system we obtain a volume part of every component. In general case the system (2.2) is solved by the iterative method. When calculating the self-shielding factor it is necessary to take into account the mutual resonance shielding of different nuclei, the resonance self-shielding by "smooth" part of cross section and Doppler effect for resolved and unresolved levels.

Fig.5 shows the sample efficiency (diameter 10cm) in the reactor PF-4 as a function of content for $\text{Re} + \text{Gd}_2\text{O}_3$ mixture. The results of calculations point at the maximum efficiency value to exist which exceeds the efficiency of samples made

of a single component. To obtain the subsequent increase of control rod efficiency the optimum mixture of three or more components has to be used.

B. Optimum composition of resonance absorber mixture to obtain the minimum depression of energy release field in thermal reactor.

The absorption cross section energy dependence is very important not only for the rod efficiency but also for a neutron flux depression near the rod. Neutron flux depression can be reduced not only by means every rod absorption decrease due to increase of the rod number but also by using the resonance absorbers as an absorption material which permit to vary the correlation between the absorption in thermal and epithermal regions. In fact the more neutron will be observed by a rod in the epithermal region the less absorption will take place in the thermal region at the fixed rod efficiency and therefore the neutron flux depression will be smaller in the thermal region, where the greatest part of fission take place. The least energy release shield depression will be in the limiting case when the total absorption takes place only in the thermal region.

As an example we consider the absorption in the plate-type rods. Using the expression $\mathcal{I} = a + b/\sqrt{N}d$ as a resonance integral (a and b constant, N - nucleus density, d - plate width) and assuming the constant "a" generally to depend on the absorption due to $1/v$ -cross section the total resonance absorption in the plate can be written

$$\mathcal{I} = l \left[\beta_0 \cdot 2\sqrt{E_0/E_{rp}} + \mathcal{E} \sqrt{\beta_0} \right] \quad (2.3)$$

where l - plate surface for a unit volume of a core, $\beta_0 = G_c^2 N d$ plate optical width for the neutron of energy $E=0.0253$ ev, E_{rp} - the energy on the boundary between Maxwellien spectrum and moderated neutron spectrum. It is clear from (2.3) that the absorption contribution caused by strong resonances can be suitably described with the value the $\mathcal{E} = \frac{b}{\sqrt{G_c}}$. This value

does not depend on the nucleus concentration and on the reactor spectrum but it is function of resonance parameters. Among all the resonance absorbers hafnium ($\xi=1.56$) tantalum ($\xi=1.4$) indium ($\xi=1.04$), antinony ($\xi=1.07$), europium ($\xi=1.02$), rhenium ($\xi=0.92$) are those which have the best correlation between absorption in the thermal and in the resonance regions. For $1/v$ -cross section this characteristic is equal to zero. Cadmium and gadolinium have the worst energy-dependence for the absorption cross section, the "smooth" part of their absorption cross sections decreasing more quickly than $1/v$ -absorption. The contribution of their epicadmium resonances is neglected and as a result the value ξ for these element should be written as negative one.

In Fig.6 the plate optical width β_0 for neutron energy $E_0=0.025\text{ev}$ is plotted as a function of value ξ which has to compensate the water reactor (ration of hydrogen nuclei to U^{235} nuclei is about 200, the square lattice pitch is 10cm). The dependence of the thermal neutron flux max-average ratio "K" for elementary cell is shown in this figure too.

Using the optimum composition mixtures we can achieve the value ξ considerably exceeding that one for a single element. In this case the optimum composition is found so as to provide the maximum resonance absorption in the plate $I = \sum_i I_i$ under extra condition $\sum_i \beta_i = \beta_0$.

If the element resonance levels are assumed to be overlapped and non-shielding absorption to be caused only by $1/v$ -cross section ($\frac{\xi_i}{\xi_k} = \text{const}$, $i=1,2,\dots,n$) this problem can be solved analytically by Lagrangian undefined multipliers method. The optimum weight content of i -th element (atomic weight A_i and thermal cross section ξ_i) will be described by the expression

$$P_i = \frac{\xi_i^2 A_i / \xi_i}{\sum_k \xi_k^2 A_k / \xi_k} \quad (2.4)$$

and maximum total resonance absorption is

$$I_{\max} = l \left[\beta_0 \cdot 2 \sqrt{\frac{E_0}{E_{2p}}} + \xi_{\max} \sqrt{\beta_0} \right] \quad \xi_{\max} = \sqrt{\sum_k \xi_k^2} \quad (2.5)$$

The optimum compositions and values ξ_{\max} for mixture of some resonance absorbers are given in the table below.

Material	In-Ag	Hf-Ta	Hf-Ta-W	Hf-Ta-Eu
P _i %	37 63	26.3 73.7	91.1 59.1 19.8	27 73 0.2
ξ_i	1.04 0.78	1.57 1.14	1.57 1.14 0.64	1.57 1.14 1.02
ξ_{\max}	1.30	1.94	2.04	2.18

REFERENCES.

- [1] A.P.Aleksandrov et al. "Atomic ice-breaker Lenin". (P/2140 Proc. 2nd UN Conf. on Atomic Energy-Geneva (1958)).
- [2] A.Radkowsky. Theory and Application of Burnable Poisons (P/1900 Proc. 2nd UN Conf. on Atomic Energy-Geneva (1958)).
- [3] V.S.Volkov et al. "Use of burnable poisons in power reactors", "Atomic energy" V.11, 1961.
- [4] The Physics of Intermedial spectrum Reactors. USAEC, 1958.
- [5] G.I.Toshinsky, A.G.Kalashnikov. "Methods for calculating burn up of absorbers in power reactors", "Theory and methods for calculation nuclear reactors". Gosatomisdat, 1962.
- [6] L.N.Ussachev, "Equation for neutron importance, reactor kinetics and perturbation theory", (P/2 Proc. 1t UN Conf. on Atomic Energy-Geneva, 1955).
- [7] I.V.Gordeev, D.A.Kardashev, A.V.Malyshev. "Iaderno-Fizicheskie constanti". Gosatomisdat, 1963.
- [8] A.I.Leipunsky et al. Experimental investigations of some physical features of intermediate spectrum reactor with berillium moderator. (P SM 18/80 International symposium on physics of fast and intermediate spectrum reactors. Vienna. 1962.).

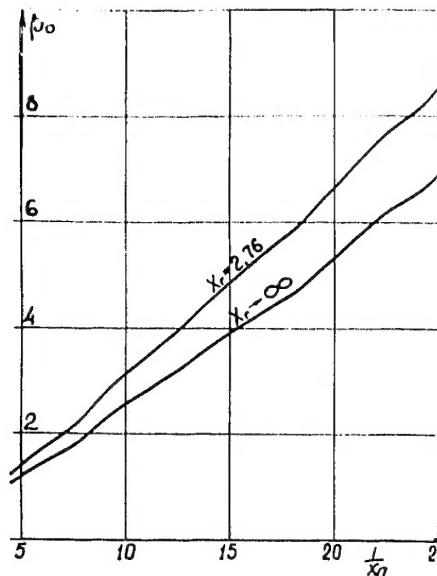


Fig. 1. The initial absorber plate optical width providing the maximum reduction of the use of mechanical control devices as a function of the absorber lifetime X_n .

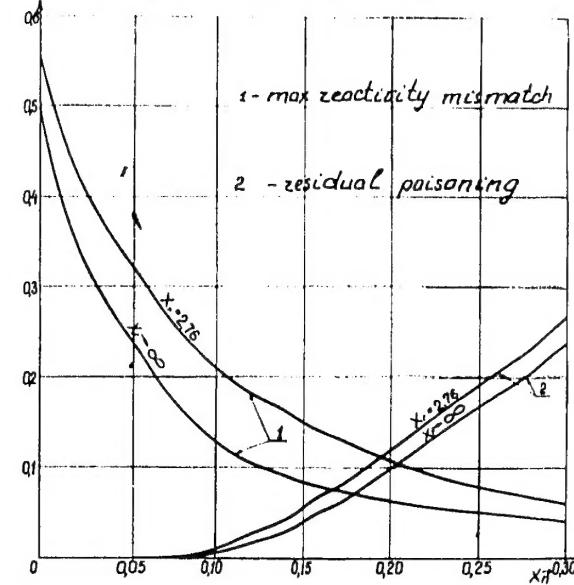


Fig. 2. The maximum reactivity mismatch ($\rho_h / \rho_{h,n}$) (1) and non-burnable residual poisoning $\rho_h'' / \rho_{h,n}''$ (2) as a function of absorber lifetime with the optical width being optimum.

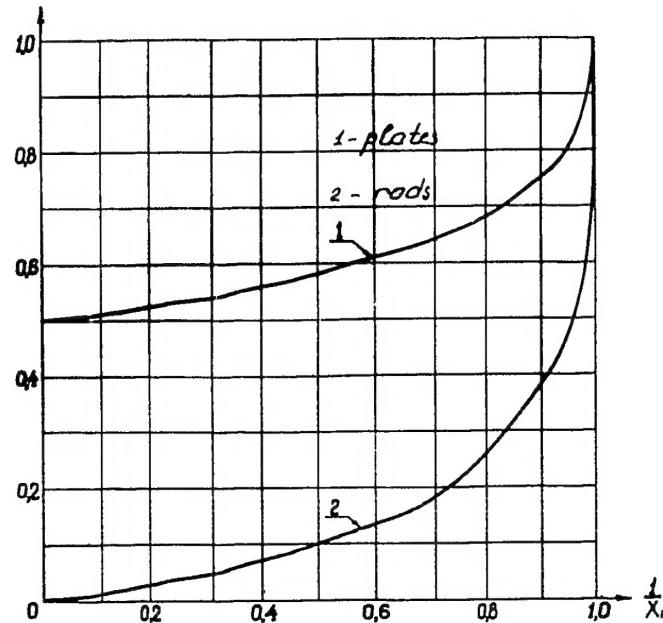


Fig. 3. The maximum reactivity mismatch for absorbers of infinite cross section in plane (1) and cylindrical geometry as a function of fuel burnup during the reactor lifetime ($1/X_r$).

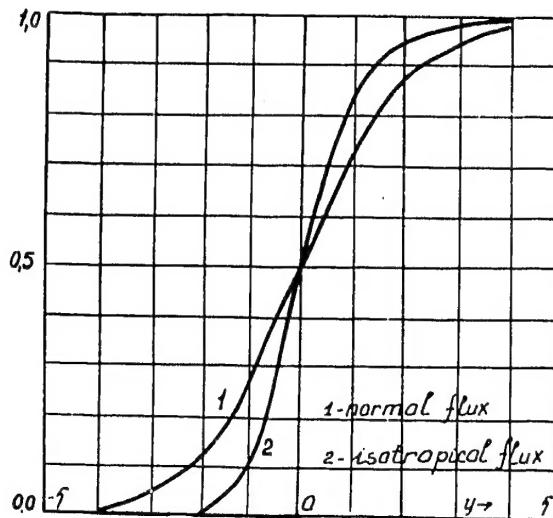


Fig. 4. The steady distribution of absorber nuclei in transient region.

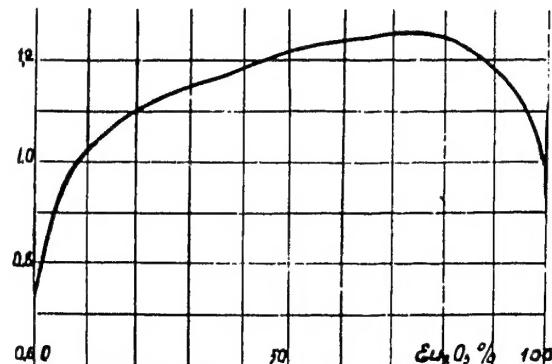


Fig. 5. The rod efficiency in the reactor NPF-4 as a function of europium oxide content in $\text{Re}-\text{Eu}_2\text{O}_3$ mixture.

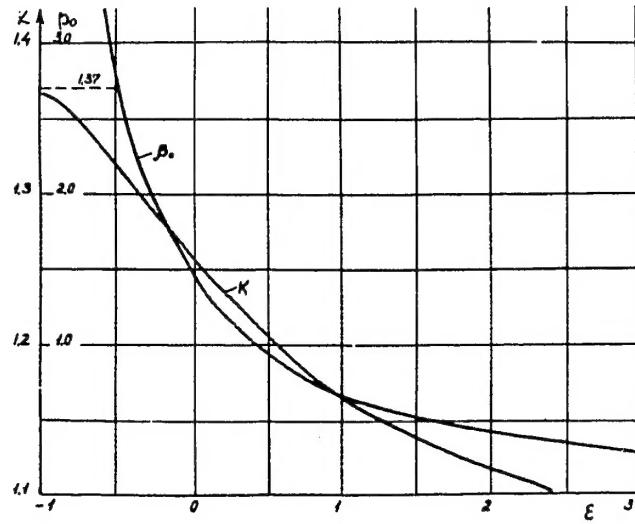


Fig. 6. Optical plate width to compensate the reactivity excess of water reactor and max-average ratio for the thermal neutron flux in its elementary as a function of ϵ .